CAVITATION MECHANISM OF HIGH-VOLTAGE BREAKDOWN FORMATION IN LIQUID DIELECTRICS

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Electrical breakdown in a liquid often plays a major role in contemporary pulsed-discharge materials processing technology. Thus, this phenomenon requires multifaceted study, and the question of formation and evolution of gas cavities and their role in discharge development is a principal one in the physics of high-voltage discharges in liquids [1, 2]. The present study will analyze the cavitation mechanism of gas cavity formation and evolution in a point-plane electrode system for the prebreakdown stage of the discharge. A comparison with experimental results of other authors [3] is presented.

We will consider the growth of a cavitation bubble generated near the point electrode. We will assume that the bubble grows because of electrostatic forces while there exists a pressure p_0 in the liquid, the pressure of ionized vapors within the bubble being small in comparison to this p_0. Moreover, we neglect the effect of the electrodes on growth dynamics, i.e., we assume that the bubble grows within an unbounded ideal incompressible liquid. Using the experimental data of [3], we assume that the bubble surface always remains equipotential with a potential equal to the electrode potential φ_0. It will be shown below that Laplacian pressure may also be neglected. With these assumptions we write the equation for the bubble radius in the form [4]

$$R\ddot{R} + (3/2)\dot{R}^2 = (p_e - p_0)/\rho,\ p_e = \varphi_0^2/(8\pi k R^2),\ k = 1/4\pi\varepsilon_0$$

(where p_e is the electrostatic pressure on the surface). With the aid of simple transformations it is easy to obtain the first integral of the equation

$$\frac{1}{2}R^3\dot{R}^2 + \frac{1}{3}\frac{p_0R^3}{\rho} - \frac{\varphi_0^2 R}{8\pi k\rho} = \text{const.}$$

Choosing as initial conditions $\dot{R} = u(t = 0) = 0$, $R(t = 0) = R_{00}$ (where R_{00} is the radius of curvature of the electrode), we obtain for the bubble growth rate an expression in dimensionless form:

$$u(R)/u_0 = \left[\frac{1}{R^3}\left(\Phi_0^2(R-1) - \frac{1}{3}(R^3-1)\right)\right]^{1/2},\quad u_0 = \sqrt{\frac{2p_0}{\rho}},\ \Phi_0^2 = \frac{\varphi_0^2}{8\pi k p_0 R_{00}^2} \tag{1}$$

(where R is measured in units of R_{00}). The function u(R) has a maximum at the point $R_m = 3/2 - 1/(2\Phi_0^2)$. It is obvious that $\Phi_0^2 > 1/3\ \max R_m = 3/2$. Aside from the point R = 1, the function u(R) vanishes at the point

$$R_1 = \sqrt{3(\Phi_0^2 - 1/4)} - 1/2. \tag{2}$$

The value of R_1 obtained here is the limiting size of an expanding spherical bubble. We will also find the limiting bubble growth rate, defining the same as $u(\max R_m) \equiv u_m$. Then

$$u_m = \frac{2}{3}u_0\left[\frac{1}{3}\left(\Phi_0^2 - \frac{19}{12}\right)\right]^{1/2}. \tag{3}$$

It is evident from Eqs. (2) and (3) that a cavitation regime of bubble growth is possible at $\Phi_0^2 > 1$. At an atmospheric pressure p_0 this yields a field intensity E > 1.8 MV/cm. Figure 1 shows functions u(R) in dimensionless units for $\Phi_0^2 = 4$ and 30.2 (curves 1 and 2). Curve 2 was constructed for n-hexane parameters and $p_0 = 10^5$ Pa, φ_0 = 33 kV, R_{00} = 0.04 mm, as used in the experiments of [3]. The maximum bubble growth rate for these parameters is ~34.8 m/sec. According to the measurements of [3] this value reaches 50 m/sec.

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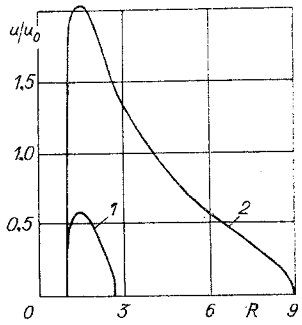

Fig. 1

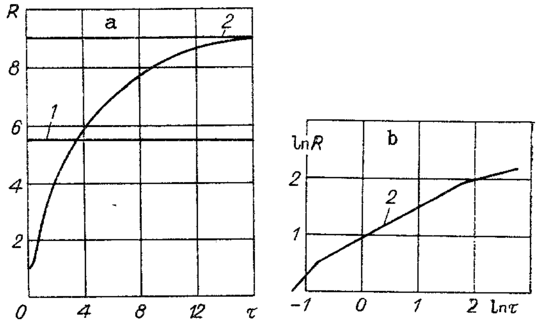

Fig. 2

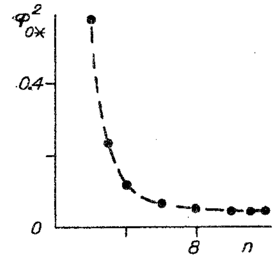

Fig. 3

To obtain the function R(t) Eq. (1) was integrated numerically. The integration results for $\Phi_0^2 = 30.2$ are shown in Fig. 2a in dimensionless variables. The dimensionless time $\tau = tu_0/R_{00}$ here. Lines 1 and 2 correspond to limiting and equilibrium values of bubble radius. The equilibrium value is determined by the equation of force (pressure) balance and is equal to $R = \Phi_0$. The dimensioned time of bubble growth to the equilibrium radius is ~8 μsec, to the limiting radius, ~38 μsec. According to the measurements of [3], the bubble doubles its initial radius over a time of ~2 μsec. Calculations with the integration results (Fig. 2a) gives a radius-doubling time of ~1.8 μsec. Figure 2b shows the function R(τ) in logarithmic scale. It is evident that in the initial growth stage $R \sim t^2$, for $1.5 < r < 7$ (segment 2) $R \sim t^{0.57}$. In [3] the experimentally determined growth exponent for segment 2 was equal to 0.6.

In [3, 5] the development of streamers in a certain stage of spherical cavity growth was related to development of instability of the liquid surface in the electric field. However no stability study was performed. In [6] in a study of a globe lightning model it was shown that equilibrium of a spherical gas cavity formed in a charged liquid shell cannot be stable with respect to perturbations of the surface grooving type.

We will analyze the stability of a spherical equilibrium cavitation cavity with charged surface in a manner analogous to that of [6]. We then assume that at the moment when instability becomes possible the perturbations increase more rapidly than the mean cavity radius. Then the state of the cavity can be considered quasiequilibrium with a radius R_0 having a potential φ_0 with constant charge $Q_0 = \varphi_0 R_0/k$. We will perform the stability study by the Rayleigh method [7], considering that, in contrast to a droplet, upon surface deformations the cavity can change its volume. We write the equation of the perturbed surface in the form

$$R = R_0\left(1 + \sum_{n=1}^{\infty} a_n P_n(\cos\theta)\right) \text{ (where } P_n \text{ are Legendre polynomials).}$$

We write the potential energy of the cavitation bubble

$$W = -\frac{1}{2} Q_0 \oint \frac{\nabla\varphi_e \, dS}{R} + \alpha \oint dS + \int p dV, \tag{4}$$

where φ_e is the potential energy in the liquid, p is the energy in the liquid, whose values satisfy the Laplace equation (in view of what has been said above, we consider the liquid immobile). The solutions of the Laplace equation satisfying conditions at infinity have the form

$$\varphi_e = \frac{\varphi_0 R_0}{r} + \varphi_0 \sum_{n=1}^{\infty} A_n \left(\frac{R_0}{r}\right)^{n+1} P_n(\cos\theta),$$

$$p = p_0 + p_0 \sum_{n=1}^{\infty} B_n \left(\frac{R_0}{r}\right)^{n+1} P_n(\cos\theta), \; r > R.$$

From the condition of surface equipotentiality $\nabla\varphi_e \tau = 0$ (where τ is a unit vector tangent to the surface) we find $A_n = a_n$. From the condition of pressure continuity $p|_R = (p_e - p_\ell)|_R$ (where p_e and p_ℓ are the electrical and Laplace pressures) we obtain

$$B_n = a_n\left(\frac{\varphi_0^2}{4\pi k p_0 R_0^2}(n-1) - \frac{\alpha}{R_0 p_0}(n^2 + n - 2)\right).$$

For stability of equilibrium it is necessary that the second variation of the potential energy be positive. Calculating the integrals in Eq. (4) to the accuracy of a_n^2 and transforming to dimensionless variables, we find the stability condition

$$\Phi_1^2 < \frac{\lambda\left[\frac{1}{3}(n-1)(n-2)(n+2)+\frac{1}{2}(n^2+n+2)\right]+1}{\left[n+\frac{2}{3}(n-1)(n-2)\right]} = \Phi_{1*}^2,$$
$$\lambda = \alpha/(p_0R_0), \quad \Phi_1^2 = \varphi_0^2/(8\pi k p_0 R_0^2). \tag{5}$$

The function $\Phi_{1*}^2(n)$ has a minimum, with n_m being the number of the most dangerous spherical harmonic. For $\lambda \ll 1$ it develops that $n_m \gg 1$ and $n_m \approx (6/\lambda)^{1/3}$, which corresponds to grooving of the cavity surface. Thus, in contrast to a charged droplet, for which the most dangerous mode with respect to stability is the ellipsoidal (n = 2), for a charged cavitation cavity within a liquid higher spherical harmonics are the most dangerous. Growth of the most dangerous perturbations can stimulate development of a finite number of streamers N. From general considerations it is clear that the perturbation growth increment is proportional to $\left(\frac{1}{m}\left|\frac{\partial^2 W}{\partial R^2}\right|\right)^{1/2}$ (where m is the reduced mass), i.e., is proportional to the square root of the potential energy variation. Since the latter is maximal for $n = n_m$, then the most dangerous perturbations have the largest increment and a number of streamers $N \approx n_m$. Considering that $\Phi_0^2R_{00}^2/R_0^2 = \Phi_1^2$ and $\lambda = \left(\frac{\alpha}{p_0R_{00}}\right)\frac{R_{00}}{R_0}$, one can calculate the critical values of the dimensionless potential Φ_{0*}^2 and n_m, using Eq. (5) and the experimental parameters of [3]. Taking, as before, R_{00} = 0.04 mm and $\alpha = 1.84\cdot10^{-2}$ N/m (n-hexane [8]), we find $\lambda = 4.6\cdot10^{-2}$ (R_{00}/R_0) and

$$\Phi_{0*}^2 = 4.3\cdot10^{-2}, \quad n_m = 11, \quad R_0 = R_{00};$$
$$\Phi_{0*}^2 = 5.5\cdot10^{-2}, \quad n_m = 12, \quad R_0 = 1.5R_{00}. \tag{6}$$

The function $\Phi_{0*}^2(n)$ for $R_0 = R_{00}$ is shown in Fig. 3. Since for formation and growth of a cavitation cavity it is necessary that $\Phi_0^2 > 1$, it is then evident from Eq. (6) that growth occurs in a regime intensely supercritical with respect to stability of spherical quasi-equilibrium. But since perturbations increase at a finite rate, then in the initial period (at least from the time of cavity formation up to $R_0 = 1.5R_{00}$) its surface may remain quasi-spherical and only subsequently do perturbations appear and streamers develop. It was observed in the experiments of [3] that at $R_0 \approx 0.2$ mm the wave number of increasing perturbations $k \approx 500$ cm^{-1}. This corresponds to $n = kR_0 = 10$. From Eq. (6) we have $n_m = 11$.

Thus, comparison of the proposed theory with experimental data permits the hope that a cavitation mechanism is responsible for formation of a high-voltage (E > 1.8 MV/cm) streamer breakdown in liquid dielectrics.

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